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Effective tachyon dynamics in superstring theory

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Abstract: A recently proposed $\ell = \infty$ field theory model of tachyon dynamics for unstable bosonic D-branes has been shown to arise as the two-derivative truncation of (boundary)-string field theory. Using an $\ell \to \infty$ limit appropriate to stable kinks we obtain a model for the tachyon dynamics on unstable D-branes or D-brane anti-D-brane pairs of superstring theory. The tachyon potential is a positive definite even function of the tachyon, and at the stable global minima there is no on-shell dynamics. The kink solution mimics nicely the properties of stable D-branes: the spectrum of the kink consists of infinite levels starting at zero mass, with spacing double the value of the tachyon mass-squared. It is natural to expect that this model will arise in (boundary) superstring field theory.

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Field theory models of tachyon dynamics have been a useful tool to understand the realization of Sen's conjectures on tachyon condensation and D-brane annihilation [1]. The p-adic string models [2], defined with a choice of prime number, describe tachyon dynamics with infinitely many spacetime derivatives and correctly show the dissappearance of on-shell dynamics at the stable vacuum. Lumps of any codimension can be found exactly. An $\ell = 3$ model [3] is a solvable model with lump solutions whose worldvolume theory can be calculated exactly, and where tachyon condensation proceeds in a way mathematically similar to the case in open string field theory. The lump spectrum contains a continuous sector, somewhat reminiscent of the closed string sector. An $\ell = \infty$ model [4] combines solvability with strikingly stringy properties. The spectrum of the lump solutions representing unstable D-branes contains a tachyon of the correct mass, and equally spaced infinite levels. There is no continuous spectrum, and nothing survives after tachyon condensation. This $\ell = \infty$ model can also be obtained as the $p \to 1$ limit in p-adic string theory [5]. In addition, the $\ell = \infty$ model can be supplemented with gauge field dynamics also showing stringy properties [4].

It was recently pointed out [5, 6] that the $\ell = \infty$ model arises as a two-derivative reduction of (boundary) string field theory (B-SFT). In particular, the tachyon potential in the model is the exact potential. B-SFT is a formally background independent approach to string field theory proposed in [7] and developed in [8]–[10]. As opposed to cubic string field theory [11], B-SFT is not precisely defined in general, but can be defined concretely for particular families of backgrounds, notably along a set of backgrounds related by relevant tachyonic deformations [9].[1] Indeed, the works of [5, 6], supplemented with the normalization calculation of [13], provide an exact verification of the energetics aspect of the tachyon conjectures. While strong evidence for the conjectures has been obtained using cubic string field theory [14] (and superstring field theory [15] for the analogous superstring conjectures) exact verification in this approach appears to require further understanding of the properties of the star algebra of open strings [16].

In this paper, following [4], we will produce a (two-derivative) field theory model for tachyon dynamics in superstring theory. Again, we will be guided by the requirements of solvability of the model, absence of dynamics on the spatially homogeneous stable vacuum, and the demand that the (stable) kink arising from the model, representing a stable D-brane, should have a string like spectrum on its worldvolume. The resulting model is

$$S = -8\mathcal{T} \int dt\, d^p x \left(\frac{1}{2} e^{-2T^2} \partial_\mu T \partial^\mu T + \frac{1}{8} e^{-2T^2} \right). \tag{1}$$

Here we are considering the tachyon field theory on the world volume of a non-BPS Dp-brane of superstring theory. It is then natural to extend this action to a

[1] For related computations in boundary CFT see [12].

coincident Dp-brane and anti-Dp-brane pair, each separately BPS. Since the tachyon field on the Dp-brane anti-Dp-brane pair is complex, in this case one must replace $T^2 \to TT^*$ and $(\partial T)^2 \to \partial T^* \, \partial T$. The tension $\mathcal{T}$ must be adjusted accordingly. The following points should be noted:

- the unstable vacuum is $T = 0$, where the tachyon mass squared is $M_T^2 = -1/2$, in units where $\alpha' = 1$.
- the potential is positive definite and an even function of T.
- the stable minima are at $T = \pm\infty$. At these points the effective mass squared of the tachyon is infinite.
- the non-BPS tachyon dynamics of (1) admits a kink (and anti-kink) solution describing a codimension one stable D-brane localized along an x coordinate. The profile is given simply by $T = \pm x/2$.
- the spectrum on the kink will be shown to consist of equally spaced mass levels $M^2 = 0, 1, 2, \ldots$. Notice that the spacing is twice the value of $|M_T^2|$, as in string theory.
- the tension $\mathcal{T}_{\text{kink}}$ satisfies the relation $(\sqrt{2}/2\pi)\,(\mathcal{T}_{\text{kink}}/\mathcal{T}) = (2/\sqrt{\pi}) \simeq 1.128$. In string theory this ratio takes the value of unity.

The fact that (1) satisfies the above properties leads us to conjecture that it will arise as the two derivative reduction of the yet to be analyzed (boundary) superstring field theory. In particular, the potential should be the exact one.

Constructing the model. Recall how the various ℓ models were constructed [17, 4]. The idea was finding a field theory admiting a stationary solution whose spectrum is governed by the Schroedinger problem with reflectionless potential[2] $U_\ell(x) = -\ell(\ell+1)\text{sech}^2 x$. For unstable lumps the ground state of the Schroedinger problem gives rise to the tachyon on the lump. The next level is associated with the translation mode of the lump, namely it equals the derivative of the profile. Once the profile is known one can readily reconstruct the potential that gives rise to it. This time, since we want a stable lump, we must identify the derivative of the profile with the ground state wavefunction. Indeed, this familiar procedure leads to the sine-Gordon soliton and the ϕ^4 kink solution for the cases $\ell = 1$ and $\ell = 2$ respectively [20]. For higher values of ℓ direct application of this procedure leads to complicated field theory potentials [20, 21]. We will focus here on the special case of interest $\ell \to \infty$, and will see that with the help of a field redefinition the action can be given in simple form.

[2]For a pedagogical review on these and other solvable Hamiltonians with references to the early literature see [18]. Applications of reflectionless systems to fermions can be found in [19].

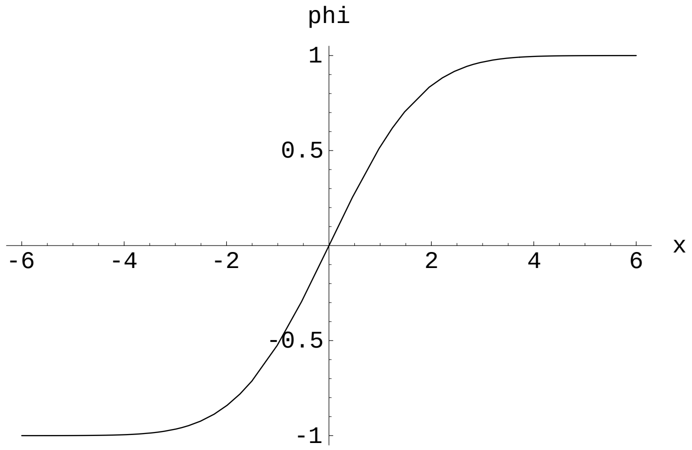

Figure 1: The profile $\overline{\phi}(x) = \text{erf}(x/2)$ of the stable kink representing a D-brane. In the field variable T used in (1) we have $\overline{T}(x) = x/2$.

The general aspects of this field redefinition, which could be used for arbitrary ℓ will be noted afterwords.

We are looking for a model taking the form

$$S \sim -\int dt d^p x \left(\frac{1}{2}\,\partial_\mu \phi\, \partial^\mu \phi + V(\phi)\right). \tag{2}$$

In the $\ell \to \infty$ limit of [4] the ground state wavefunction becomes a gaussian $\sim \exp(-x^2/4)$. We therefore identify this wavefunction with the spatial derivative of the profile $\overline{\phi}(x)$ of the stable lump solution of the model to be found. Thus, we set

$$\overline{\phi}'(x) = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4}\right), \tag{3}$$

where the constants have been chosen for convenience. It follows by integration that

$$\overline{\phi}(x) = \text{erf}\left(\frac{x}{2}\right), \qquad \text{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du\,, \tag{4}$$

where 'erf' is the familiar error function. A plot of this profile is shown in figure 1. It is a kink.

From the equation of motion for the soliton

$$\frac{1}{2}(\overline{\phi}'(x))^2 = V(\overline{\phi}(x))\,, \tag{5}$$

and from equations (3) and (4) it follows that

$$V(\overline{\phi}(x)) = \frac{1}{2\pi} \exp\left(-\frac{x^2}{2}\right) = \frac{1}{2\pi} \exp\left(-2[\,\text{erf}^{-1}(\overline{\phi})]^2\right). \tag{6}$$

JHEP03(2001)038

In the above erf $^{-1}$ denotes the inverse function to erf. While the above expression is implicit, it can be used, for example, to find an expansion for small ϕ:

$$V(\phi) = \frac{1}{2\pi} - \frac{1}{4}\phi^2 + \frac{\pi}{48}\phi^4 + \cdots . \tag{7}$$

The mass squared of the tachyon field in the field theory model is readily recognized to be $M_T^2 = -1/2$. The fluctuation spectrum is also easily obtained. Recall that the Schroedinger potential for the fluctuations is $V''(\overline{\phi}(x))$ and that $V'(\overline{\phi}(x))$ satisfies

$$\overline{\phi}''(x) = V'(\overline{\phi}(x)) . \tag{8}$$

Taking a derivative on each side of (8), we have

$$\overline{\phi}'''(x) = V''(\overline{\phi}(x))\overline{\phi}'(x) . \tag{9}$$

But by construction, $\overline{\phi}'(x)$ is the ground state solution to the harmonic oscillator equation. Thus we have

$$V''(\overline{\phi}(x)) = -\frac{1}{2} + \frac{x^2}{4} . \tag{10}$$

Note that this also gives us an easy way of computing $V''(0)$, and hence the mass of the tachyon in the open string vacuum, without actually having to invert the error function.

Thus we have obtained (once more) the potential for the simple harmonic oscillator. The Schroedinger equation determining the masses m^2 of the modes living on the kink is

$$-\frac{d^2}{dx^2}\psi(x) + \left(-\frac{1}{2} + \frac{x^2}{4}\right)\psi(x) = m^2\psi(x) , \tag{11}$$

and the masses of the fields are therefore

$$m^2 = n , \qquad n \geq 0 . \tag{12}$$

Thus, we get a massless field and equally spaced massive fields. The spacing between adjacent mass levels is twice the value of $|M_T^2|$. Note also that when $\overline{\phi} \to \pm 1$, $x \to \pm\infty$, and $V''(\overline{\phi}) \to +\infty$. This is the statement that the tachyon acquires infinite mass on the vacuum $\phi = \pm 1$.

The nature of the model is better appreciated after a field redefinition. Let

$$\phi = \text{erf}\,(T) \quad \to \quad \partial\phi = \frac{2}{\sqrt{\pi}}\, e^{-T^2}\,\partial T . \tag{13}$$

Since erf $^{-1}(\phi) = T$ we also have from (6) that

$$V(T) = \frac{1}{2\pi} e^{-2T^2} . \tag{14}$$

The kinetic term in the T variable reads

$$\frac{4}{\pi} e^{-2T^2} (\partial T)^2 \,. \tag{15}$$

We have therefore recovered the action given in (1). Finally, since $T = \mathrm{erf}^{-1}(\phi)$, the profile $\overline{T}(x)$ for the kink follows directly from (4) and is given as

$$\overline{T}(x) = \frac{x}{2} \,. \tag{16}$$

Evaluation of the kink energy using the action (1) gives

$$E = 8\mathcal{T} \int d^{p-1}y dx \left(\frac{1}{2} e^{-2\overline{T}^2} \partial_x \overline{T} \partial_x \overline{T} + \frac{1}{8} e^{-2\overline{T}^2} \right) = 2\sqrt{2\pi}\mathcal{T} \left(\int d^{p-1}y \right), \tag{17}$$

giving $\mathcal{T}_{\text{kink}} = 2\sqrt{2\pi}\,\mathcal{T}$. We therefore have

$$\frac{\sqrt{2}}{2\pi} \frac{\mathcal{T}_{\text{kink}}}{\mathcal{T}} = \frac{2}{\sqrt{\pi}} \simeq 1.128 \,. \tag{18}$$

In string theory this ratio takes the value of unity.[3]

The results found for (boundary) bosonic SFT [5, 6], together with the above results suggest that in (boundary) superstring field theory one may use tachyonic backgrounds of the type $T(X) = a + uX$. The vacuum with no brane would be at $(a, u) = (\infty, 0)$ and the kink solution would correspond to $(a, u) = (0, \infty)$. For the effective model, the kink is obtained with finite u, namely $u = 1/2$. It is therefore pleasing to see that the tension of the kink, as estimated in the model, is larger than the correct value in string theory, though, happily, not by much. It is also satisfying to see that the ratio in (18) would have been less than 1, were it not for the extra factor of $\sqrt{2}$ arising from the non-BPS brane tension.

Reconstructing potentials from profiles. As one can see from the above discussion, the reconstruction of the field theory from the kink profile is aided by a field redefinition. This field redefinition makes the kinetic term more complicated but allows a closed form solution. In general, assume one is given a kink profile

$$\overline{\phi}(x) = \mathcal{P}(x) \,, \tag{19}$$

where $\mathcal{P}(x)$ is either a monotonically increasing or a monotonically decreasing function of the coordinate $x \in [-\infty, \infty]$, as appropriate for a kink solution. It then follows that

$$V(\overline{\phi}(x)) = \frac{1}{2} (\mathcal{P}'(x))^2 \,. \tag{20}$$

[3]Recall that the tension for a non-BPS Type IIA Dp brane has an extra factor of $\sqrt{2}$ as compared to the tension for a type-IIB BPS Dp brane.

Introduce now a new field T via

$$\phi = \mathcal{P}(T) \quad \rightarrow \quad T = \mathcal{P}^{-1}(\phi)\,. \tag{21}$$

With this field variable the profile of the kink is just $\overline{T}(x) = x$. The monotonicity of $\mathcal{P}$ ensures that the relation between ϕ and T is single valued, with T defined on the full real line. Then

$$V(\overline{\phi}(x)) = \frac{1}{2}\Big(\mathcal{P}'(\,\mathcal{P}^{-1}[\,\overline{\phi}\,]\,)\Big)^2 = \frac{1}{2}\Big(\mathcal{P}'(\overline{T})\Big)^2\,, \tag{22}$$

showing that the potential is simple in terms of the new field variable. Since $\partial\phi = \mathcal{P}'(T)\partial T$ the final model is just

$$S = -\int dtdx\Big(\mathcal{P}'(T)\Big)^2\left(\frac{1}{2}(\partial T)^2 + \frac{1}{2}\right). \tag{23}$$

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JHEP03(2001)038